Preprint typeset in JHEP style - HYPER VERSION

ACT-02-11, MIFPA-11-05

Realistic Standard Model Fermion Mass Relations in Generalized Minimal Supergravity (GmSUGRA)

Csaba Balazs[1], **Tianjun Li**[2,3], **Dimitri V. Nanopoulos**[2,4,5], **Fei Wang**[1]

[1] *School of Physics, Monash University, Melbourne Victoria 3800, Australia*
[2] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA*
[3] *Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China*
[4] *Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA*
[5] *Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece*

Abstract: Grand Unified Theories (GUTs) usually predict wrong Standard Model (SM) fermion mass relation $m_e/m_\mu = m_d/m_s$ toward low energies. To solve this problem, we consider the Generalized Minimal Supergravity (GmSUGRA) models, which are GUTs with gravity mediated supersymmetry breaking and higher dimensional operators. Introducing non-renormalizable terms in the super- and Kähler potentials, we can obtain the correct SM fermion mass relations in the $SU(5)$ model with GUT Higgs fields in the **24** and **75** representations, and in the $SO(10)$ model. In the latter case the gauge symmetry is broken down to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, to flipped $SU(5) \times U(1)_X$, or to $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$. Especially, for the first time we generate the realistic SM fermion mass relation in GUTs by considering the high-dimensional operators in the Kähler potential.

Contents

1. Introduction

Supersymmetry naturally solves the gauge hierarchy problem in the Standard Model (SM). The unification of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings in the supersymmetric SM (SSM) at about 2×10^{16} GeV [1] strongly suggests the existence of a Grand Unified Theory (GUT). In addition, supersymmetric GUTs, such as the $SU(5)$ [2] and $SO(10)$ [3] models, give us deep insights into the problems of the SM such as charge quantization, the origin of many free parameters, the SM fermion masses and mixings, and beyond. Although supersymmetric GUTs are attractive, it is challenging to test them at the Large Hadron Collider (LHC), the future International Linear Collider (ILC), or other experiments.

In the traditional SSMs, supersymmetry is broken in the hidden sector, and supersymmetry breaking effects can be mediated to the observable sector via gravity [4], gauge interactions [5, 6], the super-Weyl anomaly [7, 8, 9], or other mechanisms. Recently, considering GUTs with gravity mediated supersymmetry breaking and higher dimensional operators [5, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] and F-theory GUTs with $U(1)$ fluxes [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34], two of us (LN) proposed the Generalized Minimal Supregravity (GmSUGRA) scenario and studied the generic gaugino mass relations as well as defined their indices [35]. We also generalized gauge and anomaly mediated supersymmetry breaking, and discussed the corresponding gaugino mass relations and their indices [36].

It is well known that one of the great successes of GUTs is the prediction of the equal Yukawa couplings at the GUT scale for the bottom (b) quark and τ lepton [37], which yields the correct mass ratio $m_b/m_\tau \sim 2.7$ at the low energy if and only if there are only three generations [38, 39]. Alas, it is also well known that GUTs with minimal Higgs content predict the wrong SM fermion mass relation $m_e/m_\mu = m_d/m_s$, which is invariant under the renormalization group equation (RGE) running due to the small Yukawa couplings of the first two generations. This problem can be solved via the Georgi-Jarlskog mechanism [40] by introducing Higgs fields in higher dimensional representations in $SU(5)$ models (For generalization for $SO(10)$ models, see Ref. [41].), via the Ellis-Gaillard mechanism [42] by introducing higher dimensional operators (For generalization in the supersymmetric models with mass generation for the first two families of the SM fermions, see Ref. [43].), or invoking supersymmetric loop effects [44]. Based on our previous work on SM fermion Yukawa couplings in GmSUGRA [45], we aim to generate the correct SM fermion mass relations in the $SU(5)$ and $SO(10)$ models.

In this paper, we briefly review GUTs and consider the general gravity mediated supersymmetry breaking. With non-renormalizable terms in the superpotential [42, 43] and Kähler potential, we can obtain the correct SM fermion mass relations $m_e m_s/m_d m_\mu \simeq 1/10$ in the $SU(5)$ model with GUT Higgs fields in the **24** and **75** representations, and in $SO(10)$ model where the gauge symmetry is broken down to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, to the flipped $SU(5) \times U(1)_X$ symmetry [46, 47, 48], or to $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$. Our approach can be considered as the generalizations of the Georgi-Jarlskog and Ellis-Gaillard mechanisms. However, we cannot get realistic SM fermion mass relations in $SO(10)$ models where the gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times$

$SU(2)_L \times SU(2)_R$ or to the George-Glashow $SU(5) \times U(1)'$ symmetry. In the traditional Pati-Salam and George-Glashow $SU(5)$ models, we predict $m_e/m_\mu = m_d/m_s$. We emphasize that we for the first time use the high-dimensional operators in the Kähler potentail to derive the realistic SM fermion mass relation in GUTs.

This paper is organized as follows. In Section 2, we briefly review four-dimensional GUTs. In Section 3, we explain general gravity mediated supersymmetry breaking. With higher dimensional operators in the super- and Kähler potential, we study the SM fermion mass relations in $SU(5)$-based models in Section 4. We consider $SO(10)$ models with higher dimensional operators in the super- and Kähler potential in Section 5 and Section 6, respectively. Section 7 contains our conclusion.

2. A Brief Review of Grand Unified Theories

In this Section we explain our conventions. In supersymmetric SMs, we denote the left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, right-handed neutrinos, and right-handed charged leptons as Q_i, U_i^c, D_i^c, L_i, N_i^c, and E_i^c, respectively. We denote one pair of Higgs doublets as H_u and H_d, which give masses to the up-type quarks/neutrinos and the down-type quarks/charged leptons, respectively. Moreover, we define $\tan\beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$, where $v_{u,d} \equiv \langle H_{u,d}^0 \rangle$ are the Higgs vacuum expectation values (VEVs).

First, we briefly review the $SU(5)$ model. We define the $U(1)_Y$ hypercharge generator in $SU(5)$ as follows

$$T_{\rm U(1)_Y} = {\rm diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right) . \tag{2.1}$$

Under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the $SU(5)$ representations are decomposed as follows

$$\begin{aligned}
\mathbf{5} &= (\mathbf{3}, \mathbf{1}, \mathbf{-1/3}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1/2}) \ , & (2.2)\\
\overline{\mathbf{5}} &= (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1/3}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{-1/2}) \ , & (2.3)\\
\mathbf{10} &= (\mathbf{3}, \mathbf{2}, \mathbf{1/6}) \oplus (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{-2/3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \ , & (2.4)\\
\overline{\mathbf{10}} &= (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{-1/6}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{2/3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{-1}) \ , & (2.5)\\
\mathbf{24} &= (\mathbf{8}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{3}, \mathbf{2}, \mathbf{-5/6}) \oplus (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{5/6}) \ . & (2.6)
\end{aligned}$$

There are three families of the SM fermions whose quantum numbers under $SU(5)$ are

$$F_i' = \mathbf{10}, \ \overline{f}_i' = \overline{\mathbf{5}}, \ N_i^c = \mathbf{1} \ , \tag{2.7}$$

where $i = 1, 2, 3$ for three families. The SM particle assignments in F_i' and $\bar{f}_i'$ are

$$F_i' = (Q_i, U_i^c, E_i^c) \ , \ \overline{f}_i' = (D_i^c, L_i) \ . \tag{2.8}$$

To break the $SU(5)$ and electroweak gauge symmetries, we introduce the adjoint Higgs and another pair of Higgs fields whose quantum numbers under $SU(5)$ are

$$\Phi' = \mathbf{24} \ , \quad h' = \mathbf{5} \ , \quad \overline{h}' = \overline{\mathbf{5}} \ , \tag{2.9}$$

where h' and $\overline{h}'$ contain the Higgs doublets H_u and H_d, respectively.

Next, we briefly review the flipped $SU(5)\times U(1)_X$ model [46, 47, 48]. The gauge group $SU(5) \times U(1)_X$ can be embedded into $SO(10)$. We define the generator $U(1)_{Y'}$ in $SU(5)$ as follows

$$T_{\mathrm{U(1)_{Y'}}} = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right). \tag{2.10}$$

The hypercharge is given by

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right). \tag{2.11}$$

The quantum numbers of the three SM fermion families under $SU(5) \times U(1)_X$ are

$$F_i = (\mathbf{10}, \mathbf{1}),\ \bar{f}_i = (\bar{\mathbf{5}}, -\mathbf{3}),\ \bar{l}_i = (\mathbf{1}, \mathbf{5}), \tag{2.12}$$

where $i = 1, 2, 3$. The particle assignments for the SM fermions are

$$F_i = (Q_i, D_i^c, N_i^c)\ ,\quad \overline{f}_i = (U_i^c, L_i)\ ,\quad \bar{l}_i = E_i^c\ . \tag{2.13}$$

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs fields whose quantum numbers under $SU(5) \times U(1)_X$ are

$$H = (\mathbf{10}, \mathbf{1})\ ,\quad \overline{H} = (\overline{\mathbf{10}}, -\mathbf{1})\ ,\quad h = (\mathbf{5}, -\mathbf{2})\ ,\quad \overline{h} = (\bar{\mathbf{5}}, \mathbf{2})\ , \tag{2.14}$$

where h and $\overline{h}$ contain the Higgs doublets H_d and H_u, respectively. The flipped $SU(5) \times U(1)_X$ model can be embedded into $SO(10)$. Under the $SU(5) \times U(1)_X$ gauge symmetry, the $SO(10)$ representations are decomposed as follows

$$\mathbf{10} = (\mathbf{5}, -\mathbf{2}) \oplus (\bar{\mathbf{5}}, \mathbf{2})\ , \tag{2.15}$$

$$\mathbf{16} = (\mathbf{10}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, -\mathbf{3}) \oplus (\mathbf{1}, \mathbf{5})\ , \tag{2.16}$$

$$\mathbf{45} = (\mathbf{24}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}) \oplus (\mathbf{10}, -\mathbf{4}) \oplus (\overline{\mathbf{10}}, \mathbf{4})\ . \tag{2.17}$$

Finally, we briefly review the Pati-Salam model. The gauge group is $SU(4)_C\times SU(2)_L\times SU(2)_R$ which can also be embedded into $SO(10)$. The quantum numbers of the three SM fermion families under $SU(4)_C \times SU(2)_L \times SU(2)_R$ are

$$F_i^L = (\mathbf{4}, \mathbf{2}, \mathbf{1})\ ,\quad F_i^{Rc} = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})\ , \tag{2.18}$$

where $i = 1, 2, 3$. The particle assignments for the SM fermions are

$$F_i^L = (Q_i, L_i)\ ,\quad F_i^{Rc} = (U_i^c, D_i^c, E_i^c, N_i^c)\ . \tag{2.19}$$

To break the Pati-Salam and electroweak gauge symmetries, we introduce one pair of Higgs fields and one bi-doublet Higgs field whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R$ are

$$\Phi = (\mathbf{4}, \mathbf{1}, \mathbf{2})\ ,\quad \overline{\Phi} = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})\ ,\quad H' = (\mathbf{1}, \mathbf{2}, \mathbf{2})\ , \tag{2.20}$$

where H' contains one pair of the Higgs doublets H_d and H_u. The Pati-Salam model can be embedded into $SO(10)$ as well. Under the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, the $SO(10)$ representations are decomposed as follows

$$\mathbf{10} = (\mathbf{6}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2}) \ , \tag{2.21}$$

$$\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) \oplus (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \tag{2.22}$$

$$\mathbf{45} = (\mathbf{15}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{6}, \mathbf{2}, \mathbf{2}) \ . \tag{2.23}$$

3. General Gravity Mediated Supersymmetry Breaking

The supegravity scalar potential can be written as follows [4]

$$V = e^G \left[G^i (G^{-1})_i^j G_j - 3 \right] + \frac{1}{2} \mathrm{Re} \left[(f^{-1})_{ab} \hat{D}^a \hat{D}^b \right] \ , \tag{3.1}$$

where D-terms are

$$\hat{D}^a \equiv - G^i (T^a)_i^j \phi_j = -\phi^{j*} (T^a)_j^i G_i \ , \tag{3.2}$$

and the Kähler function G as well as its derivatives and the metric G_i^j are

$$G \equiv K + \ln(W) + \ln(W^*) \ , \tag{3.3}$$

$$G^i = \frac{\delta G}{\delta \phi_i} \ , \quad G_i = \frac{\delta G}{\delta \phi_i^*} \ , \quad G_i^j = \frac{\delta^2 G}{\delta \phi_i^* \delta \phi_j} \ , \tag{3.4}$$

where K is the Kähler potential and W is the superpotential.

Since the gaugino masses, supersymmetry breaking scalar masses and trilinear soft terms have been studied previously [35], we only consider the SM fermion mass relations in this paper. We consider the following Kähler potential

$$K = \phi_i^\dagger e^{2gV} \phi_i + \frac{b_{\Phi\phi_i}}{M_*} \phi_i^\dagger e^{2gV} \Phi \phi_i + \frac{b_{S\phi_i}}{M_*} S \phi_i^\dagger e^{2gV} \phi_i \ , \tag{3.5}$$

and superpotential

$$W = \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k + \frac{1}{6} \alpha_\Phi^{ijk} \frac{\Phi}{M_*} \phi_i \phi_j \phi_k \ , \tag{3.6}$$

where M_* is the fundamental scale, Φ is the GUT Higgs field, and S is a SM singlet Higgs field.

After the scalar components of the chiral superfields Φ and S acquire vacuum expectation values (VEVs), we get the general superpotential and Kähler potential

$$K = a_{0\phi_i} \phi_i^\dagger e^{2gV} \phi_i + \frac{b_{\Phi\phi_i}}{M_*} \phi_i^\dagger \langle \Phi \rangle e^{2gV} \phi_i \ , \tag{3.7}$$

$$W = \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k + \frac{1}{6} \alpha_\Phi^{ijk} \frac{\langle \Phi \rangle}{M_*} \phi_i \phi_j \phi_k \ , \tag{3.8}$$

where

$$a_{0\phi_i} \;=\; 1 + b_{S\phi_i}\frac{\langle S\rangle}{M_*} ~. \tag{3.9}$$

Because S is a SM singlet, it can acquire a VEV close to the fundamental scale M_* . Thus, $\langle S\rangle/M_*$ can be close to 1 in principle. In short, the realistic SM fermion mass relations can be produced via these non-renormalization terms in the superpotential and Kähler potential [42, 43]. In particular, for the first time we obtain the correct SM fermion mass relation in GUTs via the high-dimensional operators in the Kähler potential.

4. $SU(5)$ Models

With non-renormalizable terms in the super- and Kähler potentials, we generate the suitable SM fermion mass ratio $m_e m_s/m_\mu m_d$ in the $SU(5)$ models. Before discussing the details, we summarize the realistic SM fermion mass relations at the GUT scale. Using low energy electroweak data, an effective universal supersymmetry breaking scale of $M_S = 500$ GeV, and two-loop RGE running for the SM gauge couplings and Yukawa couplings, we obtain the SM fermion mass ratios at the GUT scale for the down-type quarks and charged leptons [50]:

$$\frac{m_b}{m_\tau} \approx 1 ~, \quad \frac{3m_s}{m_\mu} \approx 0.69 ~, \quad \frac{m_d}{3m_e} \approx 0.83 ~. \tag{4.1}$$

Due to the small Yukawa couplings this leads to the following RGE running invariant SM fermion mass relation for the first two generations

$$\frac{m_e}{m_\mu} \approx \frac{1}{10.8}\frac{m_d}{m_s} ~. \tag{4.2}$$

For comparison, standard mass ratios at the GUT scale are [40]

$$3m_e \approx m_d ~, \quad m_\mu \approx 3m_s ~, \quad m_\tau \approx m_b ~, \tag{4.3}$$

which gives the RGE running invariant SM fermion mass ratio

$$\frac{m_e}{m_\mu} \approx \frac{1}{9}\frac{m_d}{m_s} ~. \tag{4.4}$$

4.1 Non-Renormalizable Terms in the Superpotential

In this subsection, we study new contributions to the SM fermion Yukawa couplings from higher dimensional operators in the superpotential. To obtain the possible higher dimensional operators for the Yukawa couplings, we need to consider the decompositions of the tensor products for the SM fermion Yukawa coupling terms [49]

$$\begin{aligned} \mathbf{10} \otimes \mathbf{10} \otimes \mathbf{5} &= (\bar{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \overline{\mathbf{50}}) \otimes \mathbf{5} \\ &= (\mathbf{1} \oplus \mathbf{24}) \oplus (\mathbf{24} \oplus \mathbf{75} \oplus \mathbf{126}) \oplus (\mathbf{75} \oplus \mathbf{175'}) ~, \end{aligned} \tag{4.5}$$

$$\mathbf{10} \otimes \bar{\mathbf{5}} \otimes \bar{\mathbf{5}} = \mathbf{10} \otimes (\overline{\mathbf{10}} \oplus \overline{\mathbf{15}}) = (\mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75}) \oplus (\mathbf{24} \oplus \overline{\mathbf{126}}) ~. \tag{4.6}$$

Because the Higgs fields in the $\mathbf{126}$, $\overline{\mathbf{126}}$ and $\mathbf{175'}$ do not have the $SU(3)_C \times SU(2)_L$ singlets [49], we do not consider them in the following discussions. Thus, we only consider the Higgs fields in the $\mathbf{24}$ and $\mathbf{75}$ representations.

(A) Higgs Field in the $\mathbf{24}$ Representation.

The VEVs of the Higgs field $\Phi_{\mathbf{24}}$ in the adjoint representation can be expressed as the following 5×5 and 10×10 matrices

$$\langle \Phi_{\mathbf{24}} \rangle \; = \; v\sqrt{\frac{3}{5}} \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right) \; , \tag{4.7}$$

$$\langle \Phi_{\mathbf{24}} \rangle \; = \; v\sqrt{\frac{3}{5}} \mathrm{diag}(\underbrace{-\frac{2}{3}, \cdots, -\frac{2}{3}}_{3}, \underbrace{\frac{1}{6}, \cdots, \frac{1}{6}}_{6}, 1) \; , \tag{4.8}$$

which are normalized to $c = 1/2$ and $c = 3/2$, respectively.

For the Higgs field $\Phi_{\mathbf{24}}$ in the $\mathbf{24}$ representation, we consider the following superpotential for the additional contributions to the SM fermion Yukawa coupling terms

$$\begin{aligned} W \supset \frac{1}{M_*} & \left(h^{Ui} \epsilon^{mnpql} (F'_i)_{mn} (F'_i)_{pq} (h')_k (\Phi_{\mathbf{24}})^k_l + h'^{Ui} \epsilon^{mnpkl} (F'_i)_{mn} (F'_i)_{pq} (h')_k (\Phi_{\mathbf{24}})^q_l \right. \\ & \left. + h^{DEi} (F'_i)_{mn} (\overline{f}'_i \otimes \overline{h}')^{ml}_{Sym} (\Phi_{\mathbf{24}})^n_l + h'^{DEi} (F'_i)_{mn} (\overline{f}'_i \otimes \overline{h}')^{ml}_{Asym} (\Phi_{\mathbf{24}})^n_l \right) \; , \end{aligned} \tag{4.9}$$

where the subscripts Sym and $Asym$ denote the symmetric and anti-symmetric products of two $\bar{\mathbf{5}}$ representations. After $\Phi_{\mathbf{24}}$ acquires a VEV, we obtain the Yukawa coupling terms in the superpotential

$$\begin{aligned} W \supset \frac{v}{M_*} \sqrt{\frac{3}{5}} & \left(-2h^{Ui} Q_i U^c_i H_u - h'^{Ui} Q_i U^c_i H_u - \frac{1}{6} h'^{DEi} Q_i D^c_i H_d - h'^{DEi} L_i E^c_i H_d \right. \\ & \left. + \frac{5}{6} h^{DEi} Q_i D^c_i H_d \right) \; . \end{aligned} \tag{4.10}$$

For simplicity, we assume that the masses of the first generation are dominanted by non-renormalizable terms, while the masses of the second generation are generated as in the usual GUTs. Then we have the following Yukawa coupling terms for the first generation

$$\mathcal{L} \supseteq -c_1 (\frac{1}{6} Q_1 D^c_1 H_d + L_1 E^c_1 H_d) + \frac{5}{6} c_2 Q_1 D^c_1 H_d \; , \tag{4.11}$$

where $c_1 \approx \sqrt{\frac{3}{5}} h'^{DE} \frac{v}{M_*}$, and $c_2 \approx \sqrt{\frac{3}{5}} h^{DE} \frac{v}{M_*}$. We choose $c_i v_d \sim \mathcal{O}(\mathrm{MeV})$ which is at the order of the electron and down quark masses. After electroweak symmetry breaking, choosing $c_2 \approx 12 c_1$, we can obtain the correct RGE running invariant SM fermion mass ratio at the GUT scale

$$\frac{m_e m_s}{m_\mu m_d} = \frac{6 c_1}{5 c_2 - c_1} \approx \frac{1}{10} \; . \tag{4.12}$$

(B) Higgs Field in the **75** Representation.

The VEV of the **75** dimensional Higgs field $\Phi^{[ik]}_{jl}$ can be written as follows [10]

$$\langle\Phi^{[ik]}_{jl}\rangle ~=~ \frac{v}{2\sqrt{3}}\left[\Delta^{[i}_{cj}\Delta^{k]}_{cl}+2\Delta^{[i}_{wj}\Delta^{k]}_{wl}-\frac{1}{2}\delta^{[i}_{j}\delta^{k]}_{l}\right] ~, \tag{4.13}$$

where

$$\Delta_c ~=~ \mathrm{diag}(~1,~1,~1,~0,~0)~,~~\Delta_w ~=~ \mathrm{diag}(~0,~0,~0,~1,~1)~. \tag{4.14}$$

We consider the following superpotential for the additional contributions to the SM fermion Yukawa coupling terms

$$\begin{aligned} W \supset & \Big(h^{Ui}\epsilon^{mnpjl}(F'_i)_{mn}(F'_i)_{pq}(h')_k\Phi^{[qk]}_{jl}+h'^{Ui}\epsilon^{jlpqk}(F'_i)_{mn}(F'_i)_{pq}(h')_k\Phi^{[mn]}_{jl} \\ & +h^{DEi}(F'_i)_{mn}(\overline{f}'_i)^p(\overline{h}')^q\Phi^{[mn]}_{pq}\Big)~. \end{aligned} \tag{4.15}$$

After $\Phi^{[ik]}_{jl}$ acquires a VEV, we obtain the Yukawa coupling terms in the superpotential

$$W \supset \frac{v}{M_*}\frac{1}{2\sqrt{3}}\left(-h'^{DEi}Q_iD^c_iH_d+3h'^{DEi}L_iE^c_iH_d\right)~. \tag{4.16}$$

Similarly to the Georgi-Jarlskog mechanism [40], we can get the realistic SM fermion mass relation. After imposing some discrete symmetry, we can generate the following superpotential

$$\begin{aligned} W \supseteq & \left(h^{DE}_{12}Q_1D^c_2H_d+h^{DE}_{12}L_1E^c_2H_d+h^{DE}_{12}Q_2D^c_1H_d+h^{DE}_{12}L_2E^c_1H_d\right) \\ & +\frac{v}{M_*}\frac{1}{2\sqrt{3}}\left(-h'^{DE}_{22}Q_2D^c_2H_d+3h'^{DE}_{22}L_2E^c_2H_d\right)~. \end{aligned} \tag{4.17}$$

For not too large $\tan\beta$ and $h'^{DE}\sim\mathcal{O}(1)$, we have $h'^{DE}v_dv/M_*\sim\mathcal{O}(10^2)$ MeV. Thus, we get the following mass matrices for $(e,~\mu)$ and $(d,~s)$ after electroweak symmetry breaking

$$\begin{array}{cc} & \begin{array}{cc} e & \mu \end{array} \\ \begin{array}{c} e \\ \mu \end{array} & \left(\begin{array}{cc} 0 & a \\ a & 3b \end{array}\right) \end{array}~, \qquad \begin{array}{cc} & \begin{array}{cc} d & s \end{array} \\ \begin{array}{c} d \\ s \end{array} & \left(\begin{array}{cc} 0 & a \\ a & b \end{array}\right) \end{array}~. \tag{4.18}$$

Diagonalizing these matrices for $a \ll b$, we can get approximately the RG invariant SM fermion mass ratio

$$\frac{m_e}{m_\mu}\approx\frac{1}{9}\frac{m_d}{m_s}~. \tag{4.19}$$

4.2 Non-Renormalizable Terms in the Kähler Potential

In this subsection, we study the new contributions to the SM fermion Yukawa couplings arising from higher dimensional operators in the Kähler potential. The realistic SM fermion mass ratios can also be produced by the non-minimal Kähler potentials. In order to construct gauge invariant higher dimensional operators, we need the decompositions of the following tensor products

$$\bar{\mathbf{5}} \otimes \mathbf{5} = \mathbf{1} \oplus \mathbf{24} \ , \tag{4.20}$$

$$\overline{\mathbf{10}} \otimes \mathbf{10} = \mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75} \ . \tag{4.21}$$

Thus, the adjoint Higgs field can give additional contributions to the kinetic terms for both F_i' and $\overline{f}_i'$, while the Higgs field in the **75** representation can only give an extra contribution to the kinetic term of F_i'.

For the non-minimal Kähler potential, the kinetic terms relevant to e, μ, d, s are

$$K \supseteq Z_{Q_i} Q_i^\dagger Q_i + Z_{L_i} L_i^\dagger L_i + Z_{E_i^c} (E_i^c)^\dagger (E_i^c) + Z_{D_i^c} (D_i^c)^\dagger (D_i^c) \ . \tag{4.22}$$

With the simple SM fermion Yukawa coupling terms for the charged leptons and down-type quarks

$$W = y_i^{DE} F_i' \bar{f}_i' \bar{h} \ , \tag{4.23}$$

we obtain their masses after electroweak gauge symmetry breaking

$$m_e^i = \frac{m_{DE}^i}{\sqrt{Z_{L^i} Z_{E_i^c}}} \ , \ \ m_d^i = \frac{m_{DE}^i}{\sqrt{Z_{Q^i} Z_{D_i^c}}} \ . \tag{4.24}$$

Here $m_{DE}^i = y_i^{DE} \langle H_d \rangle$ are universal for the down-type quarks and charged leptons in each generations. In this work, we assume that each normalization factor Z_Φ is positive.

(A) Higgs Field in the **24** Representation.

The VEVs of the Higgs field $\Phi_{\mathbf{24}}$ in the adjoint representation are given in Eqs. (4.7) and (4.8). Thus, we obtain the following normalizations for the SM fermion kinetic terms

$$Z_{Q_i} = a_0 + \sqrt{\frac{3}{5}} \frac{1}{6} \epsilon_1^i, \tag{4.25}$$

$$Z_{U_i} = a_0 - \sqrt{\frac{3}{5}} \frac{2}{3} \epsilon_1^i, \tag{4.26}$$

$$Z_{E_i^c} = a_0 + \sqrt{\frac{3}{5}} \epsilon_1^i, \tag{4.27}$$

$$Z_{D_i^c} = a_0' + \sqrt{\frac{3}{5}} \frac{1}{3} \epsilon_1'^i, \tag{4.28}$$

$$Z_{L_i} = a_0' - \sqrt{\frac{3}{5}} \frac{1}{2} \epsilon_1'^i, \tag{4.29}$$

where

$$a_0 = 1 + b_{S\mathbf{10}} \frac{\langle S \rangle}{M_*} \ , \qquad \epsilon_1^i = b_{\Phi\mathbf{10}}^i \frac{\langle \Phi_{\mathbf{24}} \rangle}{M_*} \ , \tag{4.30}$$

$$a'_0 = 1 + b_{S\overline{\mathbf{5}}} \frac{\langle S \rangle}{M_*} ~, \qquad \epsilon_1'^i = b^i_{\Phi\overline{\mathbf{5}}} \frac{\langle \Phi_{\mathbf{24}} \rangle}{M_*} ~, \tag{4.31}$$

where i is the family index.

Thus, we can obtain the correct SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1+\frac{1}{6})(b'_1+\frac{1}{3})(b_2+1)(b'_2-\frac{1}{2})}{(b_1+1)(b'_1-\frac{1}{2})(b_2+\frac{1}{6})(b'_2+\frac{1}{3})}} \approx \frac{1}{10} ~. \tag{4.32}$$

Here we normalize

$$a_0 = b_i \sqrt{\frac{3}{5}} \epsilon_1^i ~, \quad a'_0 = b'_i \sqrt{\frac{3}{5}} \epsilon_1'^i ~, \tag{4.33}$$

with no summation on the family index i. For instance, we can choose $b_1 \approx b_2$, $b'_1 \neq \frac{1}{2}$, while $b'_2 \approx \frac{1}{2}$.

(B) Higgs Field in the **75** Representation.

Next, we consider the Higgs field $\Phi^{[ij]}_{kl}$ in the **75** representation. Because the Higgs fields $\Phi_{\mathbf{24}}$ and $\Phi^{[ij]}_{kl}$ belong to the decomposition of the tensor product $\overline{\mathbf{10}} \times \mathbf{10}$, their VEVs must be orthogonal to each other. Thus, we obtain the VEV of $\Phi^{[ij]}_{kl}$ in terms of the 10×10 matrix

$$\langle \Phi^{[ij]}_{kl} \rangle ~=~ \frac{v}{2\sqrt{3}} \mathrm{diag} \left(\underbrace{1, \cdots, ~1}_{3}, \underbrace{-1, \cdots, -1}_{6}, 3 \right) ~. \tag{4.34}$$

So we obtain the normalizations for the SM fermion kinetic terms

$$Z_{Q_i} ~=~ a_0 - \frac{1}{2\sqrt{3}} \epsilon_3^i, \tag{4.35}$$

$$Z_{U_i} ~=~ a_0 - \frac{1}{2\sqrt{3}} \epsilon_3^i, \tag{4.36}$$

$$Z_{E^c_i} ~=~ a_0 + \frac{3}{2\sqrt{3}} \epsilon_3^i, \tag{4.37}$$

$$Z_{D^c_i} ~=~ Z_{L_i} ~=~ a_0 ~, \tag{4.38}$$

where

$$a_0 = 1 + b_{S\mathbf{10}} \frac{\langle S \rangle}{M_*} ~, \qquad \epsilon_3^i = b^i_{\Phi\mathbf{10}} \frac{\langle \Phi_{\mathbf{75}} \rangle}{M_*} ~, \tag{4.39}$$

and i denotes the family index. The realistic SM fermion mass ratio emerges as

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1-1)(b_2+3)}{(b_2-1)(b_1+3)}} \approx \frac{1}{10} ~. \tag{4.40}$$

Here we normalize

$$a_0 = b_i \frac{1}{2\sqrt{3}} \epsilon_3^i ~, \tag{4.41}$$

with no summation on the family index i. For example, we can choose $b_2 \neq 1$ while $b_1 \approx 1$.

5. $SO(10)$ Models with Non-Renormalizable Superpotential Terms

In the $SO(10)$ model, the gauge symmetry can be broken directly down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$, the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry, the Geogi-Glashow $SU(5) \times U(1)'$, and the flipped $SU(5) \times U(1)_X$ symmetry. For the last two cases, the gauge symmetry can be further reduced to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry. In the Pati-Salam models and Georgi-Glashow $SU(5) \times U(1)'$ models without further gauge symmetry breaking, the masses for the down-type quarks and charged leptons are the same. Thus, we cannot obtain the correct SM fermion mass relations when we break the $SO(10)$ gauge symmetry down to the $SU(4)_C \times SU(2)_L \times SU(2)_R$ or $SU(5) \times U(1)'$ symmetries. To be concrete, we shall also study these two scenarios in details.

There are several kinds of the renormalizable Yukawa coupling terms for the SM fermions in the $SO(10)$ models. For example, we can introduce the Higgs fields in the **120** or **126** representation to obtain additional contributions to the SM fermion Yukawa couplings. In this paper, we only consider the simplest Higgs fields[1] $H^i_{\mathbf{10}}$ $(i = 1, 2)$ in the $SO(10)$ fundamental representation. The renormalizable terms in superpotential give the tree-level mass relations

$$m_{d^i} = m_{e^i} \ , \quad m_{u^i} = m_{\nu^i} \ , \tag{5.1}$$

after the Higgs fields $H^i_{\mathbf{10}}$ acquire VEVs. Due to the arbitrariness in neutrino sector, we will not discuss the mass ratios for u^i and ν^i here. We only consider the SM fermion mass ratio $m_e m_s / m_\mu m_d$.

There are several ways to improve such mass ratio. For example, one can introduce additional higher representation Higgs fields to generalize the Georgi-Jarlskog mechanism in $SU(5)$ models [40] and Georgi-Nanopoulos mechanism in the $SO(10)$ models [41]. In this work, we generate the realistic SM fermion mass ratio in the GmSUGRA, *i.e.* in the simple $SO(10)$ model with higher dimensional operators in the super- and Kähler potentials. In this Section, we discuss the effects of non-renormalizable terms in the superpotential on the SM fermion mass relations.

To obtain the non-renormalizable contributions to the SM fermion Yukawa coupling terms, we need to know the decompositions of the tensor product $\mathbf{16} \otimes \mathbf{16} \otimes \mathbf{10}$ [49]

$$\mathbf{16} \otimes \mathbf{16} \, = \, \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \ , \tag{5.2}$$

$$\mathbf{16} \otimes \mathbf{16} \otimes \mathbf{10} \, = \, (\mathbf{1} \oplus \mathbf{45} \oplus \mathbf{54}) \oplus (\mathbf{45} \oplus \mathbf{210} \oplus \mathbf{945}) \oplus (\mathbf{210} \oplus \mathbf{1050}) \ . \tag{5.3}$$

Because the **945** and **1050** representations do not have $SU(5) \times U(1)$ or $SU(4)_C \times SU(2)_L \times SU(2)_R$ singlets [49], we only consider the Higgs fields in the **45**, **54** and **210** representations.

5.1 The Pati-Salam Model

The $SO(10)$ gauge symmetry can be broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ symmetry by giving VEVs to the Higgs fields in the **54** and **210** representations.

[1] We use two **10** Higgs to avoid large $\tan \beta$.

We can write the VEV of the Higgs field $\Phi_{\mathbf{54}}$ as

$$\langle \Phi_{\mathbf{54}} \rangle = \frac{v}{2\sqrt{15}} \text{diag}(\underbrace{2, \cdots, 2}_{6}, \underbrace{-3, \cdots, -3}_{4}) , \tag{5.4}$$

which is normalized to $c = 1$.

To calculate the additional contributions to the Yukawa coupling terms, we consider the following superpotential

$$W \supset \frac{1}{M_*} h^i (\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}} (\Phi_{\mathbf{54}})_{mn} \mathbf{10}^n . \tag{5.5}$$

After $\Phi_{\mathbf{54}}$ acquires a VEV, we obtain the additional contributions to the SM fermion Yukawa coupling terms

$$W \supset -h^i \frac{3v}{\sqrt{15} M_*} \left[Q_i U_i^c H_u + L_i N_i^c H_u + Q_i D_i^c H_d + L_i E_i^c H_d \right] . \tag{5.6}$$

Thus, the extra contributions to all the SM fermion Yukawa couplings are the same, and then we cannot explain the SM fermion mass ratio.

The VEV of the $\Phi_{\mathbf{210}}$ Higgs field can be written as

$$\langle \Phi_{\mathbf{210}} \rangle = \frac{v}{2\sqrt{2}} \text{diag}(\underbrace{1, \cdots, 1}_{8}, \underbrace{-1, \cdots, -1}_{8}) , \tag{5.7}$$

which is normalized to $c = 2$. We consider the following superpotential

$$W \supset \frac{1}{M_*} \left[h^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}} (\Phi_{\mathbf{210}})_{mnlk} \mathbf{10}^k + h'^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnlpq}_{\mathbf{126}} (\Phi_{\mathbf{210}})_{mnlp} \mathbf{10}_q \right] . \tag{5.8}$$

It is easy to show that the above superpotential will not contribute to the SM fermion Yukawa coupling terms.

In short, we cannot obtain the realistic SM fermion mass relation since the Pati-Salam gauge symmetry is not broken. This problem can be solved by introducing additional renormalizable Yukawa coupling terms involving the higher representation Higgs fields.

5.2 The $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Model

The $SO(10)$ gauge symmetry can also be broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry by giving VEVs to the $(\mathbf{15}, \mathbf{1}, \mathbf{1})$ components of the Higgs fields in the $\mathbf{45}$ and $\mathbf{210}$ representations under $SU(4)_C \times SU(2)_L \times SU(2)_R$.

For the Higgs field $\Phi_{\mathbf{45}}$ in the $\mathbf{45}$ representation, the VEV can be written as

$$\langle \Phi_{\mathbf{45}} \rangle = \frac{v}{2\sqrt{6}} \text{diag}(\underbrace{2, \cdots, 2}_{3}, \underbrace{-2, \cdots, -2}_{3}, \underbrace{0, \cdots, 0}_{4}) , \tag{5.9}$$

which is normalized as $c = 1$.

To calculate the additional contributions to the SM fermion Yukawa coupling terms, we consider the following superpotential

$$W \supset \frac{1}{M_*} \left[h^i (\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}} (\Phi_{\mathbf{45}})_{mn} \mathbf{10}^n + h'^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}} (\Phi_{\mathbf{45}})_{mn} \mathbf{10}_l \right] . \tag{5.10}$$

However, the above superpotential will not contribute to the SM fermion Yukawa coupling terms.

For the Higgs field $\Phi_{\mathbf{210}}$ in the $\mathbf{210}$ representation, the VEV is

$$\langle\Phi_{\mathbf{210}}\rangle \;=\; \frac{v}{2\sqrt{6}}\mathrm{diag}(\underbrace{1,\ 1,\ 1,-3}_{4})\ , \tag{5.11}$$

with normalization $c=2$. We consider the following superpotential

$$W \supset \frac{1}{M_*}\left[h^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{210}})_{mnlk}\mathbf{10}^k + h'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnlpq}_{\mathbf{126}}(\Phi_{\mathbf{210}})_{mnlp}\mathbf{10}_q\right] \tag{5.12}$$

After $\Phi_{\mathbf{210}}$ acquires a VEV, we obtain the additional contributions to the SM fermion Yukawa coupling terms

$$W \supset h'^i\frac{v}{\sqrt{6}M_*}\left[Q_iU_i^cH_u - 3L_iN_i^cH_u + Q_iD_i^cH_d - 3L_iE_i^cH_d\right]\ . \tag{5.13}$$

Similar to the Georgi-Jarlskog mechanism in $SU(5)$ models [40] and Georgi-Nanopoulos mechanism in $SO(10)$ models [41], we can explain the SM fermion mass ratio. After imposing some discrete symmetries, we can generate the following superpotential

$$\begin{aligned} W \supseteq\ & \left(h_{12}^{DE}Q_1D_2^cH_d + h_{12}^{DE}L_1E_2^cH_d + h_{12}^{DE}Q_2D_1^cH_d + h_{12}^{DE}L_2E_1^cH_d\right) \\ &+ \frac{v}{M_*}\frac{1}{\sqrt{6}}\left(h'^{DE}_{22}Q_2D_2^cH_d - 3h'^{DE}_{22}L_2E_2^cH_d\right). \end{aligned} \tag{5.14}$$

Again, with not too large $\tan\beta$ and $h'^{DE} \sim \mathcal{O}(1)$, we have $h'^{DE}_{22}v_dv/M_* \sim \mathcal{O}(10^2)$ MeV. Thus, we get the mass matrices for $(e,\ \mu)$ and $(d,\ s)$ after electroweak symmetry breaking

$$\begin{array}{cc} & \begin{array}{cc} e & \mu \end{array} \\ \begin{array}{c} e \\ \mu \end{array} & \left(\begin{array}{cc} 0 & a \\ a & 3b \end{array}\right) \end{array}\ , \qquad \begin{array}{cc} & \begin{array}{cc} d & s \end{array} \\ \begin{array}{c} d \\ s \end{array} & \left(\begin{array}{cc} 0 & a \\ a & b \end{array}\right) \end{array}\ . \tag{5.15}$$

Diagonalizing the mass matrices for $a \ll b$, we can get approximately the RGE running invarian SM fermion mass ratio

$$\frac{m_e}{m_\mu} \sim \frac{1}{9}\frac{m_d}{m_s}\ . \tag{5.16}$$

5.3 The Georgi-Glashow $SU(5)\times U(1)'$ Model

The $SO(10)$ gauge symmetry can be broken down to the Georgi-Glashow $SU(5)\times U(1)'$ symmetry by giving VEVs to the Higgs fields in the $\mathbf{45}$ and $\mathbf{210}$ representations.

For the Higgs field $\Phi_{\mathbf{45}}$ in the $\mathbf{45}$ representation, we can write the VEV in terms of the $\mathbf{10}\times\mathbf{10}$ matrix

$$\langle\Phi_{\mathbf{45}}\rangle \;=\; \frac{v}{\sqrt{10}}\mathrm{diag}(\underbrace{1,\cdots,\ 1}_{5},\underbrace{-1,\cdots,-1}_{5})\ , \tag{5.17}$$

where the normalization is $c = 1$. Using the conventions in [51] we obtain the non-zero components

$$(\Phi_{\mathbf{45}})_{12} = (\Phi_{\mathbf{45}})_{34} = (\Phi_{\mathbf{45}})_{56} = (\Phi_{\mathbf{45}})_{78} = (\Phi_{\mathbf{45}})_{90} = \frac{v}{\sqrt{10}} \ . \tag{5.18}$$

To calculate the additional contributions to the SM fermion Yukawa couplings, we consider the following superpotential

$$W \supset \frac{1}{M_*}\left[h^i(\mathbf{16_i}\otimes\mathbf{16_i})^m_{\mathbf{10}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}^n + h'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}_l\right] \ . \tag{5.19}$$

Note that $\mathbf{120}$ is anti-symmetric representation, the h'^i term will not contribute to the SM fermion Yukawa couplings. After $\Phi_{\mathbf{45}}$ acquires a VEV, we obtain the additional contributions to the Yukawa couplings

$$W \supset h^i\frac{2v}{\sqrt{10}M_*}\left[Q_iU^c_iH_u + L_iN^c_iH_u - Q_iD^c_iH_d - L_iE^c_iH_d\right] \ . \tag{5.20}$$

These terms are the same for the down-type quarks and charged leptons, so we cannot realize the correct SM fermion mass ratio.

For the Higgs field $\Phi_{\mathbf{210}}$ in the $\mathbf{210}$ representation, we can write the VEV in terms of the $\mathbf{16}\times\mathbf{16}$ matrix as follows

$$\langle\Phi_{\mathbf{210}}\rangle \ = \ \frac{v}{2\sqrt{5}}\mathrm{diag}(\underbrace{1,\cdots,\ 1}_{5},\underbrace{-1,\cdots,-1}_{10},5) \ , \tag{5.21}$$

where the normalization is $c = 2$. This VEV can be written in components as follows

$$\begin{aligned}(\Phi_{\mathbf{210}})_{1234} &= (\Phi_{\mathbf{210}})_{1256} = (\Phi_{\mathbf{210}})_{1278} = (\Phi_{\mathbf{210}})_{1290} = (\Phi_{\mathbf{210}})_{3456} = (\Phi_{\mathbf{210}})_{3478} \\ &= (\Phi_{\mathbf{210}})_{3490} = (\Phi_{\mathbf{210}})_{5678} = (\Phi_{\mathbf{210}})_{5690} = (\Phi_{\mathbf{210}})_{7890} = -\frac{v}{2\sqrt{5}} \ .\end{aligned} \tag{5.22}$$

We consider the following superpotential

$$W \supset \frac{1}{M_*}\left[h^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{210}})_{mnlk}\mathbf{10}^k + h'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnlkp}_{\mathbf{126}}(\Phi_{\mathbf{210}})_{mnlk}\mathbf{10}_p\right] \tag{5.23}$$

After $\Phi_{\mathbf{210}}$ acquires a VEV, we obtain the additional contributions to the SM fermion Yukawa couplings

$$W \supset h'^i\frac{v}{\sqrt{5}M_*}\left[3L_iN^c_iH_u - Q_iU^c_iH_u\right] \ . \tag{5.24}$$

In summary, we cannot obtain the realistic SM fermion mass relations in this case since the $SU(5)$ gauge symmetry is not broken. This problem can be solved by introducing additional renormalizable Yukawa coupling terms involving the higher representation Higgs fields.

5.4 The Flipped $SU(5) \times U(1)_X$ Model

The discussion for the flipped $SU(5)\times U(1)_X$ model is similar to that of the Georgi-Glashow $SU(5) \times U(1)'$ model except that we make the following transformations

$$Q_i \leftrightarrow Q_i \ , \quad U_i^c \leftrightarrow D_i^c \ , \quad L_i \leftrightarrow L_i \ , \quad N_i^c \leftrightarrow E_i^c \ , \quad H_d \leftrightarrow H_u \ . \tag{5.25}$$

Therefore, for the Higgs field in the **45** representation, we obtain the additional contributions to the SM fermion Yukawa couplins

$$W \supset h^i \frac{2v}{\sqrt{10}M_*} \left[Q_i D_i^c H_d + L_i E_i^c H_d - Q_i U_i^c H_u - L_i N_i^c H_u\right] \ . \tag{5.26}$$

These contributions are the same for the down-type quarks and charged leptons, we cannot realize the correct SM fermion mass ratio.

For the Higgs field in the **210** representation, we have

$$W \supset h'^i \frac{v}{\sqrt{5}M_*} \left[3 L_i E_i^c H_d - Q_i D_i^c H_d\right] \ . \tag{5.27}$$

Similarly to the Georgi-Jarlskog and Georgi-Nanopoulos mechanisms or to our previous discussion, we can generate the following correct SM fermion mass ratio

$$\frac{m_e}{m_\mu} \sim \frac{1}{9}\frac{m_d}{m_s} \ . \tag{5.28}$$

5.5 The $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ Model

The $SO(10)$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry by giving VEVs to the $(\mathbf{24}, \mathbf{0})$ component of the Higgs fields in the **45**, **54** and **210** representations under $SU(5) \times U(1)$, or to the $(\mathbf{75}, \mathbf{0})$ component of the Higgs field in the **210** representation. In this subsection, we will study the SM fermion Yukawa couplings in the $SO(10)$ model where the gauge symmetry is broken down to the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ symmetry via the Georgi-Glashow $SU(5) \times U(1)'$ symmetry. We also comment on the SM fermion Yukawa couplings in the $SO(10)$ model where the gauge symmetry is broken down to the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ symmetry via the flipped $SU(5) \times U(1)_X$ symmetry, which can be obtained from the Georgi-Glashow $SU(5) \times U(1)'$ case by making the replacements in Eq. (5.25).

First, for the Higgs field $\Phi_{\mathbf{45}}$ in the **45** representation, we can write the VEV in terms of the $\mathbf{10} \times \mathbf{10}$ matrix as follows

$$\langle \Phi_{\mathbf{45}} \rangle \ = \ v\sqrt{\frac{3}{5}} \mathrm{diag}(\ \frac{1}{3}, \ \frac{1}{3}, \ \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \ \frac{1}{2}, \ \frac{1}{2}) \ , \tag{5.29}$$

which is normalized to $c = 1$. It can also be written in components as follows

$$3(\Phi_{\mathbf{45}})_{12} = 3(\Phi_{\mathbf{45}})_{34} = 3(\Phi_{\mathbf{45}})_{56} = -2(\Phi_{\mathbf{45}})_{78} = -2(\Phi_{\mathbf{45}})_{90} = v\sqrt{\frac{3}{5}} \ . \tag{5.30}$$

To calculate the additional contributions to the SM fermion Yukawa couplings, we consider the following superpotential

$$W \supset \frac{1}{M_*} \left[h^i (\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{10}}^m (\Phi_{\mathbf{45}})_{mn} \mathbf{10}^n + h'^i (\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{120}}^{mnl} (\Phi_{\mathbf{45}})_{mn} \mathbf{10}_l \right] \ . \tag{5.31}$$

After $\Phi_{\mathbf{45}}$ acquires a VEV, we obtain additional contributions to the SM fermion Yukawa couplings

$$W \supset h^i \frac{v}{2M_*}\sqrt{\frac{3}{5}}\left[Q_i U_i^c H_u + L_i N_i^c H_u - Q_i D_i^c H_d - L_i E_i^c H_d\right] ~. \qquad (5.32)$$

Since these terms are universal, we cannot obtain the correct SM fermion mass ratio, and the same result holds for the intermediate flipped $SU(5)\times U(1)_X$ model.

Second, for the Higgs field $\Phi_{\mathbf{54}}$ in the $\mathbf{54}$ representation, we can write the VEV in the $\mathbf{10}\times\mathbf{10}$ matrix form as follows

$$\langle\Phi_{\mathbf{54}}\rangle ~=~ v\sqrt{\frac{3}{5}}\mathrm{diag}(~\frac{1}{3},~\frac{1}{3},~\frac{1}{3},-\frac{1}{2},-\frac{1}{2},~\frac{1}{3},~\frac{1}{3},~\frac{1}{3},-\frac{1}{2},-\frac{1}{2})~, \qquad (5.33)$$

which is normalized to $c=1$. We consider the following superpotential

$$W \supset \frac{1}{M_*} h^i (\mathbf{16_i}\otimes\mathbf{16_i})^m_{\mathbf{10}} (\Phi_{\mathbf{54}})_{mn}\mathbf{10}^n ~. \qquad (5.34)$$

After $\Phi_{\mathbf{54}}$ acquires a VEV, we obtain the additional contributions to the SM fermion Yukawa couplings

$$W \supset -h^i \frac{v}{2M_*}\sqrt{\frac{3}{5}}\left[Q_i U_i^c H_u + L_i N_i^c H_u + Q_i D_i^c H_d + L_i E_i^c H_d\right] ~. \qquad (5.35)$$

Once again, we cannot get the realistic SM fermion mass ratio, and the same result holds for the intermediate flipped $SU(5)\times U(1)_X$ model.

Third, we consider that the $(\mathbf{24},\mathbf{0})$ component of the Higgs field $\Phi^{\mathbf{24}}_{\mathbf{210}}$ in the $\mathbf{210}$ representation obtains a VEV. We can write its VEV in the $\mathbf{16}\times\mathbf{16}$ matrix as follows

$$\langle\Phi^{\mathbf{24}}_{\mathbf{210}}\rangle ~=~ \frac{v}{\sqrt{5}}\mathrm{diag}(-1,-1,-1,~\frac{3}{2},~\frac{3}{2},\underbrace{\frac{1}{6},\cdots,~\frac{1}{6}}_{6},-\frac{2}{3},-\frac{2}{3},-\frac{2}{3},~1,~0)~, \qquad (5.36)$$

which is normalized to $c=2$. In components we have

$$\begin{aligned}
6(\Phi^{\mathbf{24}}_{\mathbf{210}})_{1278} &= 6(\Phi^{\mathbf{24}}_{\mathbf{210}})_{3478} = 6(\Phi^{\mathbf{24}}_{\mathbf{210}})_{5678} = 6(\Phi^{\mathbf{24}}_{\mathbf{210}})_{1290} \\
&= 6(\Phi^{\mathbf{24}}_{\mathbf{210}})_{3490} = 6(\Phi^{\mathbf{24}}_{\mathbf{210}})_{5690} = -\frac{3}{2}(\Phi^{\mathbf{24}}_{\mathbf{210}})_{1234} \\
&= -\frac{3}{2}(\Phi^{\mathbf{24}}_{\mathbf{210}})_{1256} = -\frac{3}{2}(\Phi^{\mathbf{24}}_{\mathbf{210}})_{3456} = (\Phi^{\mathbf{24}}_{\mathbf{210}})_{7890} = \frac{v}{\sqrt{5}} ~. \qquad (5.37)
\end{aligned}$$

We consider the following superpotential

$$W \supset \frac{1}{M_*}\left[h^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi^{\mathbf{24}}_{\mathbf{210}})_{mnlk}\mathbf{10}^k + h'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnlpq}_{\mathbf{126}}(\Phi^{\mathbf{24}}_{\mathbf{210}})_{mnlp}\mathbf{10}_q\right] \qquad (5.38)$$

After $\Phi^{\mathbf{24}}_{\mathbf{210}}$ acquires a VEV, the additional contributions to the SM fermion Yukawa couplings are

$$W \supset h'^i \frac{v}{M_*}\frac{1}{6\sqrt{5}}\left[-3Q_i U_i^c H_u + 9L_i N_i^c H_u - 5Q_i D_i^c H_d + 15 L_i E_i^c H_d\right] ~. \qquad (5.39)$$

Thus, similarly to the Georgi-Jarlskog mechanism, we can realize the correct SM fermion mass ratio. The same result holds for the intermediate flipped $SU(5) \times U(1)_X$ model.

Finally, we consider that the $(\mathbf{75}, \mathbf{0})$ component of the Higgs field $\Phi^{\mathbf{75}}_{\mathbf{210}}$ in the $\mathbf{210}$ representation obtains a VEV. We can write this VEV in the $\mathbf{16} \times \mathbf{16}$ matrix form as follows

$$\langle \Phi^{\mathbf{75}}_{\mathbf{210}} \rangle \; = \; \frac{v}{3} \mathrm{diag}(\, 0,\ 0,\ 0,\ 0,\ 0, \underbrace{-1, \cdots, -1}_{6},\ 1,\ 1,\ 1,\ 3,\ 0)\ , \tag{5.40}$$

which is normalized to $c = 2$. In components we have

$$\begin{aligned}
(\Phi^{\mathbf{75}}_{\mathbf{210}})_{1278} &= (\Phi^{\mathbf{75}}_{\mathbf{210}})_{3478} = (\Phi^{\mathbf{75}}_{\mathbf{210}})_{5678} = (\Phi^{\mathbf{75}}_{\mathbf{210}})_{1290} \\
&= (\Phi^{\mathbf{75}}_{\mathbf{210}})_{3490} = (\Phi^{\mathbf{75}}_{\mathbf{210}})_{5690} = -(\Phi^{\mathbf{75}}_{\mathbf{210}})_{1234} \\
&= -(\Phi^{\mathbf{75}}_{\mathbf{210}})_{1256} = -(\Phi^{\mathbf{75}}_{\mathbf{210}})_{3456} = -\frac{1}{3} (\Phi^{\mathbf{75}}_{\mathbf{210}})_{7890} = -\frac{v}{3}\ .
\end{aligned} \tag{5.41}$$

We consider the following superpotential

$$W \supset \frac{1}{M_*} \left[h^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}} (\Phi^{\mathbf{75}}_{\mathbf{210}})_{mnlk} \mathbf{10}^k + h'^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnlpq}_{\mathbf{126}} (\Phi^{\mathbf{75}}_{\mathbf{210}})_{mnlp} \mathbf{10}_q \right] \tag{5.42}$$

After $\Phi^{\mathbf{75}}_{\mathbf{210}}$ acquires a VEV, we obtain the additional contributions to the Yukawa couplings

$$W \supset h'^i \frac{v}{3M_*} \left[-Q_i D^c_i H_d + 3 L_i E^c_i H_d \right]\ . \tag{5.43}$$

Again, similar to the Georgi-Jarlskog and Georgi-Nanopoulos mechanisms, we can obtain the correct SM fermion mass ratio. However, in this case, we cannot get the realistic SM fermion mass ratio in the intermediate flipped $SU(5) \times U(1)_X$ model.

6. $SO(10)$ Models with Non-Renormalizable Terms in the Kähler Potential

In this Section, we shall study the new contributions to the SM fermion Yukawa couplings from higher dimensional operators in the Kähler potential in the $SO(10)$ model. Normalizing the Yukawa couplings

$$W = \sum_{ab,i=1}^{2} y^{iDE}_{ab} (\mathbf{16})^a (\mathbf{16})^b (\mathbf{10}_i)\ , \tag{6.1}$$

we obtain the masses for the charged leptons and down-type quarks after electroweak symmetry breaking, which are given in Eq. (4.24).

In order to construct gauge invariant higher dimensional operators in the Kähler potential, we need to decompose the tensor product of $\overline{\mathbf{16}} \otimes \mathbf{16}$ as follows

$$\overline{\mathbf{16}} \otimes \mathbf{16} = \mathbf{1} \oplus \mathbf{45} \oplus \mathbf{210}\ . \tag{6.2}$$

Thus, we only need to consider Higgs fields in the $\mathbf{45}$ and $\mathbf{210}$ representations. The $SO(10)$ gauge symmetry can be broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$

symmetry by the VEV of the Higgs field in the **210** representation, and can be further broken to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry by the VEVs of the $(\mathbf{15}, \mathbf{1}, \mathbf{1})$ components of the Higgs fields in the **45** and **210** representations under $SU(4)_C \times SU(2)_L \times SU(2)_R$. In addition, the $SO(10)$ gauge symmetry can be broken down to the Georgi-Glashow $SU(5) \times U(1)'$ and flipped $SU(5) \times U(1)_X$ symmetries by Higgs fields in the **45** and **210** representations, and can be further broken to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ gauge symmetries by the VEV of the $(\mathbf{24}, \mathbf{0})$ component of the Higgs field in the **45** representation under $SU(5) \times U(1)$, or by the VEVs of the $(\mathbf{24}, \mathbf{0})$ and $(\mathbf{75}, \mathbf{0})$ components of the Higgs fields in the **210** representation. Thus, in the following, we consider all these gauge symmetry breaking chains.

6.1 The Pati-Salam Model

Decomposing the $\overline{\mathbf{16}} \otimes \mathbf{16}$ tensor product of spinor representations under the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, we obtain the VEV for the $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ component of the **210** dimensional Higgs field $\Phi_{\mathbf{210}}$ in terms of the 16×16 matrix

$$\langle \Phi_{\mathbf{210}} \rangle \ = \ \frac{v}{2\sqrt{2}} \mathrm{diag}(\underbrace{1, \cdots, \ 1}_{8}, \underbrace{-1, \cdots, -1}_{8}) \ , \tag{6.3}$$

with the normalization $c = 2$. This leads to the wave function normalization of the SM fermions

$$\begin{aligned} Z_{Q_i} &= a_0 + \frac{1}{2\sqrt{2}} \beta'^i_{\mathbf{210}} \frac{v}{M_*} \ , \\ Z_{U^c_i} &= a_0 - \frac{1}{2\sqrt{2}} \beta'^i_{\mathbf{210}} \frac{v}{M_*} \ , \\ Z_{E^c_i} &= a_0 - \frac{1}{2\sqrt{2}} \beta'^i_{\mathbf{210}} \frac{v}{M_*} \ , \\ Z_{D^c_i} &= a_0 - \frac{1}{2\sqrt{2}} \beta'^i_{\mathbf{210}} \frac{v}{M_*} \ , \\ Z_{L_i} &= a_0 + \frac{1}{2\sqrt{2}} \beta'^i_{\mathbf{210}} \frac{v}{M_*} \ . \end{aligned} \tag{6.4}$$

From these, we cannot obtain the suitable SM fermion mass ratio.

6.2 The $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Model

The $SO(10)$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry by giving VEVs to the $(\mathbf{15}, \mathbf{1}, \mathbf{1})$ components of the Higgs fields in the **45** and **210** representations under $SU(4)_C \times SU(2)_L \times SU(2)_R$. The decomposition of **16** under the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry is

$$\mathbf{16} = (\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1/6}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{-1/2}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{2}}, \mathbf{-1/6}) \oplus (\mathbf{1}, \mathbf{1}, \bar{\mathbf{2}}, \mathbf{1/2}) \ . \tag{6.5}$$

First, we consider the Higgs field $\Phi_{\mathbf{45}}$ in the **45** representation. The VEV of $\Phi_{\mathbf{45}}$ can be written in terms of the 16×16 matrix as follows

$$\langle \Phi_{\mathbf{45}} \rangle \ = \ \frac{v}{2\sqrt{6}} \mathrm{diag}(\underbrace{1, \ 1, \ 1, -3}_{2}, \underbrace{-1, -1, -1, \ 3}_{2}) \ , \tag{6.6}$$

which is normalized as $c = 2$. Then, the wave function normalization for the SM fermions is

$$\begin{aligned}
Z_{Q_i} &= a_0 + \frac{1}{2\sqrt{6}}\beta'^{i}_{\mathbf{45}}\frac{v}{M_*} ~, \\
Z_{U_i^c} &= a_0 - \frac{1}{2\sqrt{6}}\beta'^{i}_{\mathbf{45}}\frac{v}{M_*} ~, \\
Z_{E_i^c} &= a_0 + \frac{3}{2\sqrt{6}}\beta'^{i}_{\mathbf{45}}\frac{v}{M_*} ~, \\
Z_{D_i^c} &= a_0 - \frac{1}{2\sqrt{6}}\beta'^{i}_{\mathbf{45}}\frac{v}{M_*} ~, \\
Z_{L_i} &= a_0 - \frac{3}{2\sqrt{6}}\beta'^{i}_{\mathbf{45}}\frac{v}{M_*} ~.
\end{aligned} \tag{6.7}$$

Thus, we can obtain the correct SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1-1)(b_1+1)(b_2-3)(b_2+3)}{(b_1+3)(b_1-3)(b_2+1)(b_2-1)}} \approx \frac{1}{10} ~. \tag{6.8}$$

Here we normalize

$$a_0 = b_i \frac{1}{2\sqrt{6}}\beta'^{i}_{\mathbf{45}}\frac{v}{M_*} ~, \tag{6.9}$$

with no summation on the family index i. For example, we can choose $b_1 \neq 3$ and $b_2 \neq 1$ while $b_2 \approx 3$.

Second, we consider the Higgs field $\Phi_{\mathbf{210}}$ in the $\mathbf{210}$ representation. The VEV of $\Phi_{\mathbf{210}}$ in terms of a 16×16 matrix is

$$\langle \Phi_{\mathbf{210}} \rangle ~=~ \frac{v}{2\sqrt{6}}\mathrm{diag}(\underbrace{1,~1,~1,-3}_{4}) ~, \tag{6.10}$$

which is normalized as $c = 2$. Thus, the wave function normalization for the SM fermions is

$$\begin{aligned}
Z_{Q_i} &= a_0 + \frac{1}{2\sqrt{6}}\beta'_{\mathbf{210}}\frac{v}{M_*} ~, \\
Z_{U_i^c} &= a_0 + \frac{1}{2\sqrt{6}}\beta'^{i}_{\mathbf{210}}\frac{v}{M_*} ~, \\
Z_{E_i^c} &= a_0 - \frac{3}{2\sqrt{6}}\beta'^{i}_{\mathbf{210}}\frac{v}{M_*} ~, \\
Z_{D_i^c} &= a_0 + \frac{1}{2\sqrt{6}}\beta'^{i}_{\mathbf{210}}\frac{v}{M_*} ~, \\
Z_{L_i} &= a_0 - \frac{3}{2\sqrt{6}}\beta'^{i}_{\mathbf{210}}\frac{v}{M_*} ~.
\end{aligned} \tag{6.11}$$

So we can obtain the realistic SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1+1)^2(b_2-3)^2}{(b_1-3)^2(b_2+1)^2}} \approx \frac{1}{10} ~. \tag{6.12}$$

Here we normalize

$$a_0 = b_i \frac{1}{2\sqrt{6}} \beta'^i_{\mathbf{210}} \frac{v}{M_*} ~, \tag{6.13}$$

with no summation on the family index i. For instance, we can choose $b_1 \neq 3$ while $b_2 \approx 3$.

6.3 The Georgi-Glashow $SU(5) \times U(1)'$ and Flipped $SU(5) \times U(1)_X$ Models

The $SO(10)$ gauge symmetry can also be broken down to the $SU(5) \times U(1)$ symmetry by the VEVs of the **45** and **210** dimensional Higgs fields $\Phi_{\mathbf{45}}$ and $\Phi_{\mathbf{210}}$. The decomposition of the **16** spinor representation under $SU(5) \times U(1)$ is

$$\mathbf{16} = (\mathbf{10},~\mathbf{1}) \oplus (\mathbf{\bar{5}},~-\mathbf{3}) \oplus (\mathbf{1},~\mathbf{5}) ~. \tag{6.14}$$

(A) Higgs Field in the **45** Representation.

First, we consider the Higgs field $\Phi_{\mathbf{45}}$. From Eq. (6.14), we obtain the VEV of $\Phi_{\mathbf{45}}$ in terms of the 16×16 matrix

$$\langle \Phi_{\mathbf{45}} \rangle ~=~ \frac{v}{2\sqrt{10}} \mathrm{diag}(\underbrace{-3, \cdots, -3}_{5}, \underbrace{1, \cdots, 1}_{10}, 5) ~, \tag{6.15}$$

which is normalized as $c=2$. Consequently, we obtain the wave function normalization in the Georgi-Glashow $SU(5) \times U(1)'$ and flipped $SU(5) \times U(1)_X$ models:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$\begin{aligned} Z(F'_i) ~&=~ a_0 + \beta'^i_{\mathbf{45}} \frac{v}{2\sqrt{10}M_*} ~, \\ Z(\overline{f}'_i) ~&=~ a_0 - 3\beta'^i_{\mathbf{45}} \frac{v}{2\sqrt{10}M_*} ~, \\ Z(N^c_i) ~&=~ a_0 + 5\beta'^i_{\mathbf{45}} \frac{v}{2\sqrt{10}M_*} ~. \end{aligned} \tag{6.16}$$

  We cannot obtain the correct SM fermion mass relation in the symmetry breaking chain from $SO(10)$ down to the Georgi-Glashow $SU(5) \times U(1)'$ gauge symmetry since $SU(5)$ is not broken.

- The Flipped $SU(5) \times U(1)_X$ Model

$$\begin{aligned} Z(F_i) ~&=~ a_0 + \beta'^i_{\mathbf{45}} \frac{v}{2\sqrt{10}M_*} ~, \\ Z(\overline{f}_i) ~&=~ a_0 - 3\beta'^i_{\mathbf{45}} \frac{v}{2\sqrt{10}M_*} ~, \\ Z(\overline{l}_i) ~&=~ a_0 + 5\beta'^i_{\mathbf{45}} \frac{v}{2\sqrt{10}M_*} ~. \end{aligned} \tag{6.17}$$

In the symmetry breaking chain from $SO(10)$ to the flipped $SU(5) \times U(1)_X$ gauge symmetry, we can get the realistic SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1+1)^2(b_2-3)(b_2+5)}{(b_2+1)^2(b_1-3)(b_1+5)}} \approx \frac{1}{10} ~. \tag{6.18}$$

Here we normalize

$$a_0 = b_i \beta'^i_{\mathbf{45}} \frac{v}{2\sqrt{10} M_*} \ , \tag{6.19}$$

with no summation on the family index i. We can choose $b_1 \neq 3$ while $b_2 \approx 3$.

(B) Higgs Field in the **210** Representation.

We consider the $\Phi_{\mathbf{210}}$ Higgs field, the VEV of which is orthogonal to that of the $\Phi_{\mathbf{45}}$

$$\langle \Phi \rangle = \frac{v}{2\sqrt{5}} \mathrm{diag}(\underbrace{1, \cdots, 1}_{5}, \underbrace{-1, \cdots, -1}_{10}, 5) \ , \tag{6.20}$$

and is normalized as $c = 2$. So we obtain the wave function normalizations for the SM fermions in the Georgi-Glashow $SU(5) \times U(1)'$ and flipped $SU(5) \times U(1)_X$ models:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$\begin{aligned} Z(F'_i) &= a_0 - \beta'^i_{\mathbf{210}} \frac{v}{2\sqrt{5} M_*} \ , \\ Z(\overline{f}'_i) &= a_0 + \beta'^i_{\mathbf{210}} \frac{v}{2\sqrt{5} M_*} \ , \\ Z(N^c_i) &= a_0 + 5\beta'^i_{\mathbf{210}} \frac{v}{2\sqrt{5} M_*} \ . \end{aligned} \tag{6.21}$$

  Thus, we cannot obtain the suitable SM fermion mass relation in the symmetry breaking chain from the $SO(10)$ gauge symmetry down to the Georgi-Glashow $SU(5) \times U(1)'$ gauge symmetry since the $SU(5)$ gauge symmetry is not broken.

- The Flipped $SU(5) \times U(1)_X$ Model

$$\begin{aligned} Z(\tilde{F}_i) &= a_0 - \beta'^i_{\mathbf{210}} \frac{v}{2\sqrt{5} M_*} \ , \\ Z(\tilde{\overline{f}}_i) &= a_0 + \beta'^i_{\mathbf{210}} \frac{v}{2\sqrt{5} M_*} \ , \\ Z(\tilde{\overline{l}}_i) &= a_0 + 5\beta'^i_{\mathbf{210}} \frac{v}{2\sqrt{5} M_*} \ . \end{aligned} \tag{6.22}$$

In the symmetry breaking chain from the $SO(10)$ gauge symmetry down to the flipped $SU(5) \times U(1)_X$ gauge symmetry, we can realize the correct SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1 - 1)^2 (b_2 + 1)(b_2 + 5)}{(b_2 - 1)^2 (b_1 + 1)(b_1 + 5)}} \approx \frac{1}{10} \ . \tag{6.23}$$

Here we normalize

$$a_0 = b_i \beta'^i_{\mathbf{210}} \frac{v}{2\sqrt{5} M_*} \ , \tag{6.24}$$

with no summation on the family index i. For instance, we can choose $b_2 \neq 1$ while $b_1 \approx 1$.

6.4 The $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ Model

The $SO(10)$ gauge symmetry can also be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry by the VEV of the $(\mathbf{24}, \mathbf{0})$ component of the Higgs field in the $\mathbf{45}$ representation under $SU(5) \times U(1)$, or by the VEVs of the $(\mathbf{24}, \mathbf{0})$ and $(\mathbf{75}, \mathbf{0})$ components of the Higgs fields in the $\mathbf{210}$ representation.

(A) Higgs Field in the $(\mathbf{24}, \mathbf{0})$ Component of the $\mathbf{45}$ Representation.

First, we consider the Higgs field $\Phi^{\mathbf{24}}_{\mathbf{45}}$ in the $\mathbf{45}$ representation whose $(\mathbf{24}, \mathbf{0})$ component acquires the following VEV

$$\langle \Phi^{\mathbf{24}}_{\mathbf{45}} \rangle \;=\; v\sqrt{\frac{3}{5}} \mathrm{diag}(\, \frac{1}{3}, \,\frac{1}{3}, \,\frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, \underbrace{\frac{1}{6}, \cdots, \,\frac{1}{6}}_{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \,1, \,0) \;, \qquad (6.25)$$

which is normalized to $c = 2$.

From this, we obtain the wave function normalizations for the SM fermions in the Georgi-Glashow $SU(5) \times U(1)'$ and flipped $SU(5) \times U(1)_X$ models:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$\begin{aligned} Z_{Q_i} &= a_0 + \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{1}{6} \frac{v}{M_*} \;, \\ Z_{U^c_i} &= a_0 - \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{2}{3} \frac{v}{M_*} \;, \\ Z_{E^c_i} &= a_0 + \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{v}{M_*} \;, \\ Z_{D^c_i} &= a_0 + \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{1}{3} \frac{v}{M_*} \;, \\ Z_{L_i} &= a_0 - \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{1}{2} \frac{v}{M_*} \;. \end{aligned} \qquad (6.26)$$

In the symmetry breaking chain from the $SO(10)$ gauge symmetry via Georgi-Glashow $SU(5) \times U(1)'$ down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry, we can get the correct SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1 + \frac{1}{3})(b_1 + \frac{1}{6})(b_2 - \frac{1}{2})(b_2 + 1)}{(b_1 - \frac{1}{2})(b_1 + 1)(b_2 + \frac{1}{3})(b_2 + \frac{1}{6})}} \approx \frac{1}{10} \;. \qquad (6.27)$$

Here we normalize

$$a_0 = b_i \beta'^{i\mathbf{24}}_{\mathbf{45}} \sqrt{\frac{3}{5}} \frac{v}{M_*} \;, \qquad (6.28)$$

with no summation on the family index i. For instance, we can choose $b_1 \neq \frac{1}{2}$ while $b_2 \approx \frac{1}{2}$.

- The Flipped $SU(5) \times U(1)_X$ Model

$$\begin{aligned} Z_{Q_i} &= a_0 + \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{1}{6} \frac{v}{M_*} \ , \\ Z_{U^c_i} &= a_0 + \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{1}{3} \frac{v}{M_*} \ , \end{aligned} \tag{6.29}$$

$$\begin{aligned} Z_{E^c_i} &= a_0 \ , \\ Z_{D^c_i} &= a_0 - \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{2}{3} \frac{v}{M_*} \ , \\ Z_{L_i} &= a_0 - \sqrt{\frac{3}{5}} \beta'^{i\mathbf{24}}_{\mathbf{45}} \frac{1}{2} \frac{v}{M_*} \ . \end{aligned} \tag{6.30}$$

In the symmetry breaking chain from the $SO(10)$ gauge symmetry via flipped $SU(5) \times U(1)_X$ down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ gauge symmetry, we can obtain the realistic SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1 - \frac{2}{3})(b_1 + \frac{1}{6})(b_2 - \frac{1}{2})b_2}{(b_1 - \frac{1}{2})b_1 (b_2 - \frac{2}{3})(b_2 + \frac{1}{6})}} \approx \frac{1}{10} \ . \tag{6.31}$$

Here we normalize

$$a_0 = b_i \beta'^{i\mathbf{24}}_{\mathbf{45}} \sqrt{\frac{3}{5}} \frac{v}{M_*} \ , \tag{6.32}$$

with no summation on the family index i. For example, we can choose $b_1 \neq \frac{1}{2}$ and $b_2 \neq \frac{2}{3}$ while $b_1 \approx \frac{2}{3}$ and/or $b_2 \approx \frac{1}{2}$.

(B) Higgs Field in the $(\mathbf{24}, \mathbf{0})$ Component of the $\mathbf{210}$ Representation.

Second, we consider the Higgs field $\Phi^{\mathbf{24}}_{\mathbf{210}}$ in the $\mathbf{210}$ representation whose $(\mathbf{24}, \mathbf{0})$ component acquires a VEV as follows

$$\langle \Phi^{\mathbf{24}}_{\mathbf{210}} \rangle \ = \ \frac{v}{\sqrt{5}} \mathrm{diag}(-1, -1, -1, \ \frac{3}{2}, \ \frac{3}{2}, \underbrace{\frac{1}{6}, \cdots, \ \frac{1}{6}}_{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \ 1, \ 0) \ , \tag{6.33}$$

which is normalized to $c = 2$. In this case the wave function normalizations for the SM fermions via the Georgi-Glashow $SU(5) \times U(1)'$ and the flipped $SU(5) \times U(1)_X$ models are:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$\begin{aligned} Z_{Q_i} &= a_0 + \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{1}{6} \frac{v}{M_*} \ , \\ Z_{U^c_i} &= a_0 - \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{2}{3} \frac{v}{M_*} \ , \end{aligned}$$

$$
\begin{aligned}
Z_{E_i^c} &= a_0 + \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{v}{M_*} ~, \\
Z_{D_i^c} &= a_0 - \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{v}{M_*} ~, \\
Z_{L_i} &= a_0 + \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{3}{2} \frac{v}{M_*} ~.
\end{aligned}
\tag{6.34}
$$

In the symmetry breaking chain from the $SO(10)$ gauge symmetry via Georgi-Glashow $SU(5) \times U(1)'$ down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry, we can get the realistic SM fermion mass ratio

$$
\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1 - 1)(b_1 + \frac{1}{6})(b_2 + \frac{3}{2})(b_2 + 1)}{(b_1 + \frac{3}{2})(b_1 + 1)(b_2 + \frac{1}{6})(b_2 - 1)}} \approx \frac{1}{10} ~. \tag{6.35}
$$

Here we normalize

$$
a_0 = b_i \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{1}{\sqrt{5}} \frac{v}{M_*} ~, \tag{6.36}
$$

with no summation on the family index i. For example, we can choose $b_2 \neq 1$ while $b_1 \approx 1$.

- The Flipped $SU(5) \times U(1)_X$ Model

$$
\begin{aligned}
Z_{Q_i} &= a_0 + \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{1}{6} \frac{v}{M_*} ~, \\
Z_{U_i^c} &= a_0 - \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{v}{M_*} ~, \\
Z_{E_i^c} &= a_0 ~, \\
Z_{D_i^c} &= a_0 - \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{2}{3} \frac{v}{M_*} ~, \\
Z_{L_i} &= a_0 + \frac{1}{\sqrt{5}} \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{3}{2} \frac{v}{M_*} ~.
\end{aligned}
\tag{6.37}
$$

In the symmetry breaking chain from the $SO(10)$ gauge symmetry via flipped $SU(5) \times U(1)_X$ down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry, we can obtain the correct SM fermion mass ratio

$$
\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1 + \frac{1}{6})(b_1 - \frac{2}{3})(b_2 + \frac{3}{2}) b_2}{(b_1 + \frac{3}{2}) b_1 (b_2 + \frac{1}{6})(b_2 - \frac{2}{3})}} \approx \frac{1}{10} ~. \tag{6.38}
$$

Here we normalize

$$
a_0 = b_i \beta'^{i\mathbf{24}}_{\mathbf{210}} \frac{1}{\sqrt{5}} \frac{v}{M_*} ~, \tag{6.39}
$$

with no summation on the family index i. For instance, we can choose $b_2 \neq \frac{2}{3}$ while $b_1 \approx \frac{2}{3}$.

(C) Higgs Field in the $(\mathbf{75}, \mathbf{0})$ Component of the $\mathbf{210}$ Representation.

Third, we consider the Higgs field $\Phi^{\mathbf{75}}_{\mathbf{210}}$ in the $\mathbf{210}$ representation whose $(\mathbf{75}, \mathbf{0})$ component acquires the following VEV

$$\langle \Phi^{\mathbf{75}}_{\mathbf{210}} \rangle ~=~ \frac{v}{3} \mathrm{diag}(~0,~0,~0,~0,~0, \underbrace{-1, \cdots, -1}_{6},~1,~1,~1,~3,~0)~, \tag{6.40}$$

which is normalized to $c = 2$. Thus, we obtain the following wave function normalizations for the SM fermions via the Georgi-Glashow $SU(5) \times U(1)'$ and the flipped $SU(5) \times U(1)_X$ models:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$\begin{aligned} Z_{Q_i} &= a_0 - \frac{1}{3} \beta^{\prime i \mathbf{75}}_{\mathbf{210}} \frac{v}{M_*}~, \\ Z_{U_i^c} &= a_0 + \frac{1}{3} \beta^{\prime i \mathbf{75}}_{\mathbf{210}} \frac{v}{M_*}~, \\ Z_{E_i^c} &= a_0 + \beta^{\prime i \mathbf{75}}_{\mathbf{210}} \frac{v}{M_*}~, \\ Z_{D_i^c} &= a_0~, \\ Z_{L_i} &= a_0~. \end{aligned} \tag{6.41}$$

In the symmetry breaking chain from the $SO(10)$ gauge symmetry via Georgi-Glashow $SU(5) \times U(1)'$ down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ symmetry, we can obtain the correct SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1 - 1)(b_2 + 3)}{(b_1 + 3)(b_2 - 1)}} \approx \frac{1}{10}~. \tag{6.42}$$

Here we normalize

$$a_0 = b_i \frac{1}{3} \beta^{\prime i \mathbf{75}}_{\mathbf{210}} \frac{v}{M_*}~, \tag{6.43}$$

with no summation on the family index i. For instance, we can choose $b_2 \neq 1$ while $b_1 \approx 1$.

- The Flipped $SU(5) \times U(1)_X$ Model

$$\begin{aligned} Z_{Q_i} &= a_0 - \frac{1}{3} \beta^{\prime i \mathbf{75}}_{\mathbf{210}} \frac{v}{M_*}~, \\ Z_{U_i^c} &= a_0~, \\ Z_{E_i^c} &= a_0~, \\ Z_{D_i^c} &= a_0 + \frac{1}{3} \beta^{\prime i \mathbf{75}}_{\mathbf{210}} \frac{v}{M_*}~, \\ Z_{L_i} &= a_0~. \end{aligned} \tag{6.44}$$

In the symmetry breaking chain from the $SO(10)$ gauge symmetry via flipped $SU(5)\times U(1)_X$ down to the $SU(3)_C\times SU(2)_L\times U(1)_1\times U(1)_2$ symmetry, we can get the realistic SM fermion mass ratio

$$\frac{m_e m_s}{m_\mu m_d} = \sqrt{\frac{(b_1-1)(b_1+1)b_2^2}{b_1^2(b_2-1)(b_2+1)}} \approx \frac{1}{10}~. \tag{6.45}$$

Here we normalize

$$a_0 = b_i \frac{1}{3}\beta_{\mathbf{210}}^{\prime i\mathbf{75}}\frac{v}{M_*}~, \tag{6.46}$$

with no summation on the family index i. For instance, we can choose $b_2 \neq 1$ while $b_1 \approx 1$.

7. Conclusion

Grand Unified Theories (GUTs) usually predict wrong Standard Model (SM) fermion mass relations, such as $m_e/m_\mu = m_d/m_s$, toward low energies. Based on our previous work on the SM fermion Yukawa couplings in the GmSUGRA scenario with the higher dimensional operators containing the GUT Higgs fields, we studied the SM fermion mass relations. Considering non-renormalizable terms in the super- and Kähler potentials, we can obtain the correct SM fermion mass relations in the $SU(5)$ model with GUT Higgs fields in the **24** and **75** representations, and in the $SO(10)$ model where the gauge symmetry is broken down to $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, to the flipped $SU(5)\times U(1)_X$ symmetry, or to $SU(3)_C\times SU(2)_L\times U(1)_1\times U(1)_2$. However, we cannot improve the SM fermion mass relations in the $SO(10)$ model if the gauge symmetry is only broken down to the Pati-Salam $SU(4)_C\times SU(2)_L\times SU(2)_R$ or the George-Glashow $SU(5)\times U(1)'$ symmetry. In particular, for the first time we generate the realistic SM fermion mass relation in GUTs by considering the high-dimensional operators in the Kähler potential.

Acknowledgments

This research was supported in part by the Australian Research Council under project DP0877916 (CB and FW), by the DOE grant DE-FG03-95-Er-40917 (TL and DVN), by the Natural Science Foundation of China under grant No. 10821504 (TL), and by the Mitchell-Heep Chair in High Energy Physics (TL).

References

[1] J. R. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **249**, 441 (1990); Phys. Lett. B **260**, 131 (1991); U. Amaldi, W. de Boer and H. Furstenau, coupling Phys. Lett. B **260**, 447 (1991); P. Langacker and M. X. Luo, Phys. Rev. D **44**, 817 (1991).

[2] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[3] H. Georgi, "Particles And Fields: Williamsburg 1974. AIP Conference Proceedings No. 23", Editor C. E. Carlson; H. Fritzsch and P. Minkowski, Annals Phys. **93**, 193 (1975); H. Georgi and D. V. Nanopoulos, Nucl. Phys. B **155**, 52 (1979).

[4] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B **119**, 343 (1982); J. R. Ellis, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **121**, 123 (1983); J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and K. Tamvakis, Supergravity," Phys. Lett. B **125**, 275 (1983); L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D **27**, 2359 (1983).

[5] J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **247**, 373 (1984).

[6] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B **189**, 575 (1981); S. Dimopoulos and S. Raby, Nucl. Phys. B **192**, 353 (1981); M. Dine and W. Fischler, Phys. Lett. B **110**, 227 (1982); M. Dine and A. E. Nelson, Phys. Rev. **D48**, 1277 (1993); M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. **D51**, 1362 (1995); M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. **D53**, 2658 (1996); for a review, see G. F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999).

[7] L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999); G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998).

[8] I. Jack and D. R. T. Jones, Phys. Lett. B **482**, 167 (2000); N. Arkani-Hamed, D. E. Kaplan, H. Murayama and Y. Nomura, JHEP **0102**, 041 (2001); R. Kitano, G. D. Kribs and H. Murayama, Phys. Rev. D **70**, 035001 (2004).

[9] A. Pomarol and R. Rattazzi, JHEP **9905**, 013 (1999); R. Rattazzi, A. Strumia and J. D. Wells, Nucl. Phys. B **576**, 3; N. Okada, Phys. Rev. **D65**, 115009 (2002).

[10] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **155**, 381 (1985).

[11] C. T. Hill, Phys. Lett. B **135**, 47 (1984).

[12] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984).

[13] M. Drees, Phys. Lett. B **158**, 409 (1985).

[14] G. Anderson, H. Baer, C. h. Chen and X. Tata, Phys. Rev. D **61**, 095005 (2000).

[15] N. Chamoun, C. S. Huang, C. Liu and X. H. Wu, Nucl. Phys. B **624**, 81 (2002).

[16] J. Chakrabortty and A. Raychaudhuri, Phys. Lett. B **673**, 57 (2009).

[17] S. P. Martin, Phys. Rev. D **79**, 095019 (2009).

[18] S. Bhattacharya and J. Chakrabortty, Phys. Rev. D **81**, 015007 (2010).

[19] D. Feldman, Z. Liu and P. Nath, Phys. Rev. D **80**, 015007 (2009).

[20] N. Chamoun, C. S. Huang, C. Liu and X. H. Wu, arXiv:0909.2374 [hep-ph].

[21] Stefan Antusch, Martin Spinrath, Phys. Rev. D**79**, 095004(2009).

[22] Joydeep Chakrabortty, Amitava Raychaudhuri, arXiv:1006.1252 [hep-ph].

[23] C. Vafa, Nucl. Phys. B **469**, 403 (1996).

[24] R. Donagi and M. Wijnholt, arXiv:0802.2969 [hep-th].

[25] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 058 (2009).

[26] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 059 (2009).

[27] R. Donagi and M. Wijnholt, arXiv:0808.2223 [hep-th].

[28] A. Font and L. E. Ibanez, JHEP **0902**, 016 (2009).

[29] J. Jiang, T. Li, D. V. Nanopoulos and D. Xie, Phys. Lett. B **677**, 322 (2009).

[30] R. Blumenhagen, Phys. Rev. Lett. **102**, 071601 (2009).

[31] J. Jiang, T. Li, D. V. Nanopoulos and D. Xie, Nucl. Phys. B **830**, 195 (2010).

[32] T. Li, Phys. Rev. D **81**, 065018 (2010).

[33] G. K. Leontaris and N. D. Tracas, arXiv:0912.1557 [hep-ph].

[34] T. Li, J. A. Maxin and D. V. Nanopoulos, arXiv:1002.1031 [hep-ph].

[35] T. Li and D. V. Nanopoulos, Phys. Lett. B **692**, 121 (2010).

[36] T. Li and D. V. Nanopoulos, arXiv:1005.3798 [hep-ph].

[37] A. J. Buras, J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **135**, 66 (1978).

[38] D. V. Nanopoulos and D. A. Ross, Nucl. Phys. B **157**, 273 (1979).

[39] D. V. Nanopoulos and D. A. Ross, Phys. Lett. B **118**, 99 (1982).

[40] H. Georgi and C. Jarlskog, Phys. Lett. B**86**,297(1979).

[41] H. Georgi and D.V. Nanopoulos, Phys. Lett. B **82**,392(1979); Nucl. Phys. B **155**,52(1979); Nucl. Phys. B **159**,16 (1979).

[42] John Ellis, Mary K. Gaillard, Phys. Lett. B**88**,315(1979).

[43] D. V. Nanopoulos and M. Srednicki, Phys. Lett. B **124**, 37 (1983).

[44] J. L. Diaz-Cruz, H. Murayama, A. Pierce, Phys. Rev. D**65**, 075011(2002).

[45] C. Balazs, T. Li, D. V. Nanopoulos and F. Wang, JHEP **1009**, 003 (2010).

[46] S. M. Barr, Phys. Lett. B **112**, 219 (1982).

[47] J. P. Derendinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B **139**, 170 (1984).

[48] I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B **194**, 231 (1987).

[49] R. Slansky, Phys. Rept. **79**, 1 (1981).

[50] Graham Ross, Mario Serna, Phys. Lett. B**664**,97(2008).

[51] X. G. He and S. Meljanac, Phys. Rev. D **41**, 1620 (1990).